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Reference Point Based Multi-Objective Optimization Using Hybrid Artificial Immune System

Waiel F. Abd El-Wahed*
Faculty of Computers & Information
Menoufia University, Shebin El-Kom, Egypt
Waeilf@yahoo.com

Elsayed M. Zaki and Adel M. El-Refaey
Faculty of Engineering
Menoufia University, Shebin El-Kom, Egypt
{elsayedzaki68, adel_elrefaey}@yahoo.com

***Abstract*—during the last decade, the field of Artificial Immune System (AIS) is progressing slowly and steadily as a branch of Computational Intelligence (CI).There has been increasing interest in the development of computational models inspired by several immunological principles. Although there are advantages of knowing the range of each objective for Pareto-optimality and the shape of the Pareto-optimal frontier itself in a problem for an adequate decision-making, the task of choosing a single preferred Pareto optimal solution is also an important task. In this paper, a Reference Point Based Multi-Objective Optimization Using hybrid Artificial intelligent approach based on the clonal selection principle of Artificial Immune System (AIS) and Neural Networks is proposed. And, instead of one solution, a preferred set of solutions near the reference points can be found. Modified Multi-objective Immune System Algorithm (MISA) is proposed with real parameters value not binary coded parameters, uniform and non uniform mutation operator is applied to the clones produced. Real parameter MISA works on continuous search space.**

Keywords: Artificial Immune System, Neural Networks, Reference point approach, interactive multi-objective method, multi-objective optimization. Clonal Selection

I. INTRODUCTION

Artificial Immune System (AIS) are a new research area that takes ideas from our biological immune system to solve complex problems, mainly in engineering and the science. From the information processing perspective, the immune system can be seen as a parallel and distributed adaptive system [2, 10]. It is capable of learning; it uses memory and is capable of associative retrieval of information in recognition and classification tasks. Particularly, it learns to recognize patterns, it remembers patterns that it has been shown in the past and its global behavior is an emergent property of many local interactions. All these features of the immune system provide, in consequence, great robustness, fault tolerance, dynamism and adaptability [11]. These are the properties of the immune system that mainly attract researchers to try to emulate it in a computer.

The Multi-objective Immune System Algorithm (MISA) can be considered as the first real proposal of MOAIS in literature (Coello Coello & Cruz Cortés, 2002). In the first proposal of the algorithm, authors attempted to follow the clonal selection principle very closely, then the algorithm performances have been improved in a successive version (Cruz Cortés & Coello Coello 2003a, 2003b; Coello Coello & Cruz Cortés, 2005) sacrificing some of the biological metaphor. The population is encoded by binary strings and it is initialized randomly. The algorithm does not use explicitly a scalar index to define the avidity of a solution but some rules are defined for choosing the set of antibodies to be cloned. The ranking scheme uses the following criteria: 1) first feasible and no dominated individuals, then 2) infeasible no dominated individuals, finally 3) infeasible and dominated. The memory set (called secondary population) is updated by the no dominated feasible individuals. Because of this repository being limited in size, an adaptive grid is implemented to enforce a uniform distribution of no dominated solutions.[4,10]

But our Modified Multi-objective Immune System Algorithm (MMISA) the population takes real value and it is initialized randomly in the range assigned by Neural Networks (NN). Only feasible no dominated individual (best antibody) added to secondary population. All individuals in secondary population are cloned and mutation operators are applied to clones.

Neural Network (NN) is a well-known as one of powerful computing tools to solve optimization problems. Due to massive computing unit neurons and parallel mechanism of neural network approach it can solve the large-scale problem efficiently and optimal solution can be obtained [9,13,15]. The other hand Neural Network (NN) approach is attended as a new method for solving optimization problems, this method has a great charm because NN can solve large scale and complex optimization problems in real time, and also is benefit to search the global solution.A general methodology for solving Multi-objective Nonlinear Programming (MONP) problems. In order to operationalize the concept of Pareto-optimal solution, we should relate it to a familiar concept. The most common strategy is to characterize Pareto optimal solutions in terms of optimal solutions of appropriate Nonlinear Programming Problems (NLPP). Among Weighted Aggregation (WA) technique we can characterize Multi-objective Programming Problems (MOPs) into NLPPs. [8, 18].

Corresponding Author: Waiel F. Abd El-Wahed, Faculty of Computers & Information, Menoufia University, Shebin El-Kom, Egypt

We run Neural networks based on weighted aggregation method, with weights to determine the end points of the Pareto front and the point that all objective functions has equal weight. From these three point we deduce the range (the upper and lower values) of each decision variable. This range used as the input to AIS this modification makes AIS faster, and give more accurate Pareto Optimal solutions.[19]

In this paper, the concept of reference point methodology is used and attempts to find a set of preferred Pareto optimal solutions near the regions of interest to a decision maker. All simulation runs on test problems and on engineering design problem show another use of a Modified MISA methodology in allowing the decision-maker to solve multi-objective optimization problems better and with more confidence.

II. The Immune Systems

The main goal of the immune system is to protect the human body from the attack of foreign (harmful) organisms. The immune system is capable of distinguishing between the normal components of our organism and the foreign material that can cause us harm (e.g. bacteria). These foreign organisms are called antigens. The molecules called antibodies play the main role on the immune system response. The immune response is specific to a certain foreign organism (antigen). When an antigen is detected, those antibodies that best recognize an antigen will proliferate b cloning. This process is called clonal selection principle, the new cloned cells undergo high rate of mutation.[4,10]

A. *Clonal Selection Theory*

Any molecule that can be recognized by the adaptive immune system is known as an Ag. When an animal is exposed to an Ag, some subpopulation of its bone-marrow-derived cells (Blymphocytes) responds by producing Ab's. Ab's are molecules attached primarily to the surface of B cells whose aim is to recognize and bind to Ag's. Each B cell secretes a single type of Ab, which is relatively specific for the Ag. By binding to these Ab's and with a second signal from accessory cells, such as the T-helper cell, the Ag stimulates the B cell to proliferate (divide) and mature into terminal (no dividing) Ab secreting cells, called plasma cells. The process of cell division (mitosis) generates a clone, i.e., a cell or set of cells that are the progenies of a single cell. B cells, in addition to proliferating and differentiating into plasma cells, can differentiate into long-lived B memory cells. Memory cells circulate through the blood, lymph, and tissues and, when exposed to a second antigenic stimulus, commence to differentiate into plasma cells capable of producing high-affinity Ab's, preselected for the specific Ag that had stimulated the primary response.[12,19]

III. Reference Point Interactive Approach

The interactive multi-objective optimization technique of Wierzbickiis is very simple and practical. Before the solution process starts, some information is given to the decision maker about the problem. The goal is to achieve a weakly, ϵ-properly or Pareto-optimal solution closest to a supplied reference point of aspiration level based on solving an achievement scalarizing problem. Given a reference point $\bar{z}$ for an k-objective optimization problem of minimizing $(f_1(x), \dots, f_k(x))$ with $x \in S$, the following single-objective optimization problem is solved for this purpose:[6]

$$\text{Minimize } max_{i=1}^{k}[w_i(f_i(x) - \bar{z}_i)]$$

$$\text{Subject to } x \in S. \quad (1)$$

Here, w_i is the i-th component of a chosen weight vector used for scalarizing the objectives. Figure 1 illustrates the concept [6]. For a chosen reference point, the closest Pareto optimal solution (in the sense of the weighted-sum of the objectives) is the target solution to the reference point method. To make the procedure interactive and useful in practice, Wierzbicki suggested a procedure in which the obtained solution $\acute{z}$ is used to create k new reference points, as follows:

$$z^{(j)} = \bar{z} + (\acute{z} - \bar{z}).\, e^{(j)} \quad (2)$$

where $e^{(j)}$ is the j-th coordinate direction vector. For the two-objective problem shown in the figure, two such new reference points (z_A and z_B) are also shown.

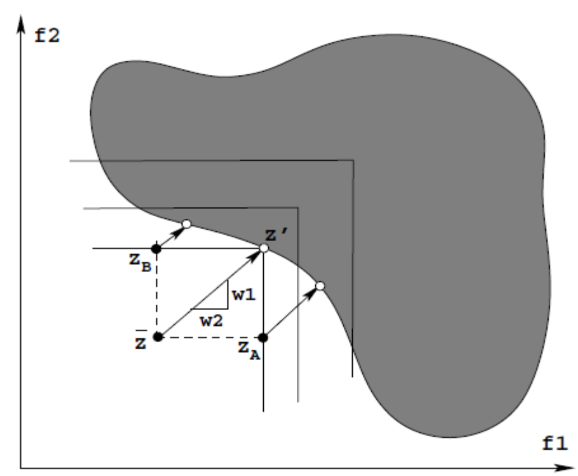

Fig. 1: Classical reference point approach. [6]

New Pareto optimal solutions are then found by forming new achievement scalarizing problems. If the decision-maker is not satisfied with any of these Pareto-optimal solutions, a new reference point is suggested and the above procedure is repeated. By repeating the procedure from different reference points, the decision-maker tries to evaluate the region of Pareto optimality, instead of one particular Pareto-optimal point. It is also interesting to note that the reference point may be a feasible one or an infeasible point which cannot be obtained from any solution from the feasible search space. If a reference point is feasible and is not a Pareto-optimal solution or infeasible, the decision-maker may then be interested in knowing solutions which are Pareto-optimal and close to the reference point. [6,8]

To utilize the reference point approach in practice, the decision-maker needs to supply a reference point and a weight vector at a time. The location of the reference point causes the procedure to focus on a certain region in the Pareto-optimal frontier, whereas a supplied weight vector makes a finer trade-off among the objectives and focuses the procedure to find a single Pareto-optimal solution (in most situations) trading-off the objectives. Thus, the reference point provides a higher-level information about the region to focus and weight vector provides a more detailed information about what point on the Pareto-optimal front to converge.[6]

IV. MULTI-OBJECTIVE PROGRAMMING (MOP) PROBLEM

This section provides the necessary mathematical background for MOP.[8,18].

Consider a Multi-objective Programming Problem with k-objectives

$(f_j(x),\ j = 1,2,\dots,k)$ and n decision variables

$(x_i,\ i = 1,2,..,n)$:

MOP:

$$\text{Min } F(x) = (f_1(x),\dots,f_k(x))$$
$$\text{subject to } S = \{x \in R^n \mid g(x) \geq 0, h(x) = 0\} \quad (3)$$

Where $x \in R^n, f : R^n \to R^k$, is k-dimensional vector valued continuous functions of n variables, $g = [g_1,\dots,g_m]^T : R^n \to R^m$, is m-dimensional vector valued continuous functions of n variables and $h = [h_1,\dots,h_p]^T : R^n \to R^p$, is p-dimensional vector valued continuous functions of n variables.[19]

The k objectives are conflicted with each other. Therefore, the target of MOP is to achieve a set of efficient solutions that are called Pareto set. The related concepts of *Pareto Optimal Solution*, and *Weak Pareto Optimal Solution* [3,18]

Definition 1(Pareto Optimal Solution): x^* is said to be Pareto optimal solution of MOP If there is no other feasible x such that**,** $\boldsymbol{f_j(x^*) \leq f_j(x)}$ for all *j*, $j = 1,2,\dots,k$ with strict inequality for at least one *j*.

Definition 2 (Weak Pareto optimal solution): $x^* \in S$ is said to be a weak Pareto optimal solution if and only if there is no other $x \in S$ such that $f_j(x^*) < f_j(x)$ for all *j*, $j = 1,2,\dots,k$ **.**

A. *Weighted Method for MOP Problem*

The weighted method [8] for the Multi-objective optimization problem is formulated as:

$$\textbf{\textit{P(w)}}: \quad Min \quad \sum_{i=1}^{k} w_i f_i(x)$$
$$s.t.\ \ x \in s,\ \ w \in W, \quad (4)$$
$$W = \{w \in R^k \mid w_j \geq 0, \sum_{j=1}^{k} w_j = 1\}$$

Multi-objective optimization runs are conducted with different weighting vector (*W*) in order to locate a set of points on the Pareto front. This method is the simplest and the most straight forward way of obtaining the Pareto-optimal front. However, this method is associated with some major drawbacks. Depending on the scaling of the different objectives and the shape of the Pareto front, it is hard to select the weighting. Another problem occurs when the solution space is non-convex. In that case not all the Pareto-optimal solutions can be obtained by solving the problem ***P(w)***. But in our study we concentrate on the end points of the Pareto Front to avoid this weakness of ***P(w)***.[8]

Theorem 1: If $x^* \in S$ is an optimal solution of the weighting problem ***P(w)*** where either $\boldsymbol{W > 0}$, or x^* is a unique optimal solution, then x^* is a Pareto optimal solution of the MOP. [8]

Theorem 2: Let the multi-objective optimization problem be convex. If $x^* \in S$ is an efficient solution of the MOP, then x^* is an optimal solution of the weighting problem ***P(w)*** for some $W = (w_1, w_2, \dots, w_k) \geq 0$. [8]

V. THE MULTI-OBJECTIVE OPTIMIZATION NEURAL NETWORK

To formulate the optimization problem in terms of a neural network, the key step is to construct an appropriate energy function *E*(*z*) such that the lowest energy state corresponds to the intended optimal solution z*. Based on the energy function, we construct a gradient system of differential equations which corresponds to a *Neural Network*.[9,20]

The (MOP) is transformed via the Weighted approach into single nonlinear Programming problem

According to the result in Rao [14], the dual Nonlinear programming problem (DNPP) can formulated as follows:

DNLPP:

$$\max_{x,\lambda,\mu} \ L(x,\lambda,\mu),\ x \in R^n$$
$$\text{s.t} \quad \nabla L(x,\lambda,\mu) = 0 \quad , \quad (5)$$
$$\lambda \geq 0, \quad \mu \text{ unrestricted in sign}$$

Where

$\lambda=(\lambda_1,\lambda_2,...,\lambda_m)^T$, $\mu=(\mu_1,\mu_2,...,\mu_p)^T$

$$L(x,\lambda,\mu)=F(x)-\sum_{i=1}^{m}\lambda_i g_i(x)-\sum_{j=1}^{p}\mu_j h_j(x)\equiv L(z)$$

$$\nabla L(x,\lambda,\mu)=\nabla F(x)-\sum_{i=1}^{m}\lambda_i\nabla g_i(x)-\sum_{j=1}^{p}\mu_j\nabla h_j(x)$$

The energy function of Convex NLP can be constructed as follows:

$$E(z)=E(x,\lambda,\mu)=\tfrac{1}{2}\left[\lambda^T g(x)\right]^2+\tfrac{1}{2}\left\|\nabla_x L(x,\lambda,\mu)\right\|^2 + \tfrac{1}{2}g(x)^T\left(g(x)-|g(x)|\right)+\tfrac{1}{2}\left\|Ax-b\right\|^2 + \tfrac{1}{2}\lambda^T(\lambda-|\lambda|) \quad (6)$$

where $z=(x^T,\lambda^T,\mu^T)^T\in R^{n+m+p}$. Every term of the right-hand side of Equation (6) being zero corresponds to every equality or inequality being satisfied in Equation (5) and $E(z)\geq 0$. Thus $E(z)$ is differentiable function by Leung et al.[9]

Theorem 3. $z^*=(x^*,\lambda^*,\mu^*)^T$ is zero point of $E(z)\Leftrightarrow z^*$ is an optimal solution of NLPP and DNLPP (i.e. x^* and $(x^*,\lambda^*,\mu^*)^T$ are optimal solutions of NLPP and DNLPP for a specific value of $\boldsymbol{W}$, respectively).

Employing the unified idea in Leung et al.[9],we can use the gradient system to construct the following multi-objective neural network for solving a convex multi-objective optimization problems:

$$\frac{dz}{dt}=-\nabla E(z) \quad (7)$$

Suppose $\nabla E(z)$ is Lipschitz continuous, then the initial value problem of differential equations in Equation (7) has a unique solution because the function in the right hand side of differential equation (7) are continuous, these equations can easily be achieved by hardware implementation of the network. Therefore, it is a feasible neural network [7,20].

VI. THE PROPOSED APPROACH

The algorithm run in two stages, the first one run Neural Networks with a random initial input based on the weighted method with three points which is the end points and the midpoint of weights. The second stage uses the output of Neural Networks as the input to MMISA which has taken ideas from the clonal selection principle, [1,4] modeling the fact that only the highest affinity antibodies with a smaller preference distance will proliferate.

For each reference point, the weighted Euclidean distance of each solution of the Pareto front is calculated and the solutions are sorted in ascending order of distance. This way, the solution closest to the reference point is assigned a rank of one.

$$d_{ij}=\sqrt{\sum_{i=1}^{n}w_i\left(\frac{f_i(x)-\bar{z}_i}{f_i^{\max}-f_i^{\min}}\right)^2} \quad (8)$$

Where $f_i^{\max}$ and $f_i^{\min}$ are the population maximum and minimum function values of *i*-th objective. Note that this weighted distance measure can also be used to find a set of preferred solutions in the case of problems having non convex Pareto-optimal front.

Using a weight vector emphasizing each objective function equally or using $w_i=1/k$. If the decision-maker is interested in biasing some objectives more than others, a suitable weight vector can be used with each reference point and solutions with a shortest weighted Euclidean distance from the reference point can be emphasized.[6]

A. . *The algorithm*

The proposed algorithm for solving Multi-objective Immune System Algorithm (MISA) based on NN is as follow:

Neural Network Simulation Algorithm[13]

[Step 1] Initialization

Let $t=0$. Randomly choose initial vector $x(t)\in \mathrm{R}^n$, $\lambda(t)\in\mathrm{R}^m$, $\mu(t)\in\mathrm{R}^p$, $\Delta t>0$ (for example $\Delta t=0.0001$) and error $\varepsilon=10^{-4}$.

[Step2] Transform the of MOP into NLPP.

[Step 3] Computation of gradient:

$$u(t)=\nabla_x E(z)=\lambda^T g(x).g(x)^T\lambda + \nabla g(x)^T[g(x)-|g(x)|] + \nabla_{xx}^2 L(z)\nabla_x L(z)+A^T(Ax-b)$$

$$v(t)=\nabla_\lambda E(z) = \lambda^T g(x).g(x)-\nabla g(x)\nabla_x L(z)+[\lambda-|\lambda|]$$

$$w(t)=\nabla_\mu E(z)=-A\nabla_x L(z)$$

[Step 4] States Updating:

$$x(t+\Delta t)=x(t)-\Delta t.u(t)$$

$$\lambda(t+\Delta t)=\lambda(t)-\Delta t.v(t)$$

$$\mu(t+\Delta t)=\mu(t)-\Delta t.w(t)$$

[Step 5] Calculate: $s=\sum_{i=1}^{n}u_i^2(t)$, $r=\sum_{j=1}^{m}v_j^2(t)$,

$$q=\sum_{j=1}^{p}w_j^2(t)$$

[Step 6] Stopping Rule:

if $s < \varepsilon, r < \varepsilon$ and $q < \varepsilon$, then output $x(t+\Delta t)$, $\lambda(t+\Delta t)$, $\mu(t+\Delta t)$ into the input file of MISA; otherwise let $t = t + \Delta t$ and go to step 3.

Proposed Reference Point Based MMISA Simulation Algorithm

[Step 1] Initialization based on NN output

[Step 2] Sorting population according to dominance

[Step 3] For each reference point, the normalized weighted Euclidean distance of each solution of the front is calculated and the solutions are sorted in ascending order of distance.

[Step 4] Choose the "best" antibodies to be cloned (feasible nondominated with shortest distance)

[Step 5] Cloning "best" antibodies

[Step 6] Appling a uniform mutation to the clones

[Step 7] Appling a non uniform mutation to some clones of antibodies

[Step 8] Repeat this process from step 2 until stopping criterion is met.

VII. Experiments

In order to validate our approach, four benchmark functions which reported in the standard evolutionary Multi-objective optimization literature.

Test Problem (1) [5]

Minimize $f_1 = x_1^2/4$

Minimize $f_2 = x_1(1-x_2)+5$

s.to $x_1, x_2 > 0$

The output of NN indicates that the range of variables will $0 \le x_1 \le 10$ and $0 \le x_2 \le 10$. Preferred solutions for three reference points $(20,-50)$,$(5,-70)$ and $(10,0)$, using Reference Point based MMISA based NN shown in fig (2)

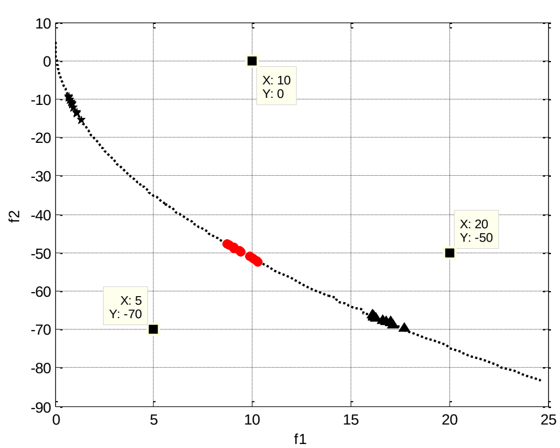

Fig. (2) Preferred solutions for three reference points.

Test Problem (2).[5]

Maximize $f_1 = 1.1 - x_1$

Maximize $f_2 = 60 - \frac{(1+x_2)}{x_1}$

s.to $0.1 \le x_1 \le 1$ and $0 \le x_2 \le 10$

The output of NN indicates that the range of variables will $0.1 \le x_1 \le 1$ and $0 \le x_2 \le 5$. Preferred solutions for three reference points$(0.5,56)$,$(0.8,52)$ and $(0.9,57)$, using Reference Point based MMISA based NN shown in fig (3)

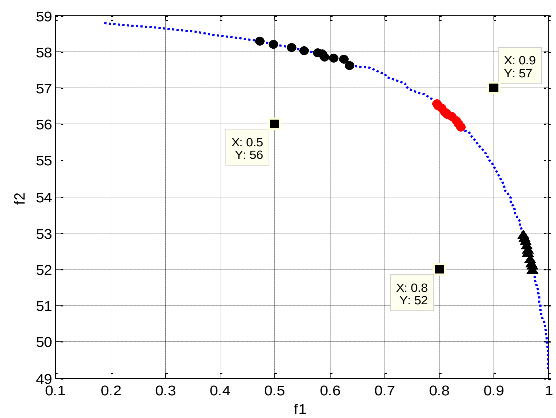

Fig. (3) Preferred solutions for three reference points.

Test Problem (3).[17]

Minimize $f_1 = x_1$

Minimize $f_2 = g(x)h(x)$

s.to $x_1, x_2 > 0$

Where $g(x) = 1 + 10\,x_2$ and

$$h(x) = 1 - \left(\frac{f_1}{g}\right)^2 - \frac{f_1}{g}\sin(8\pi f_1)$$

The output of NN indicates that the range of variables will $0 \le x_1 \le 1$ and $0 \le x_2 \le 1$. Preferred solutions for three reference points$(0.2,0.8)$,$(0.6,0.4)$ and $(0.7,-0.2)$, using Reference Point based MMISA based NN shown in fig (4)

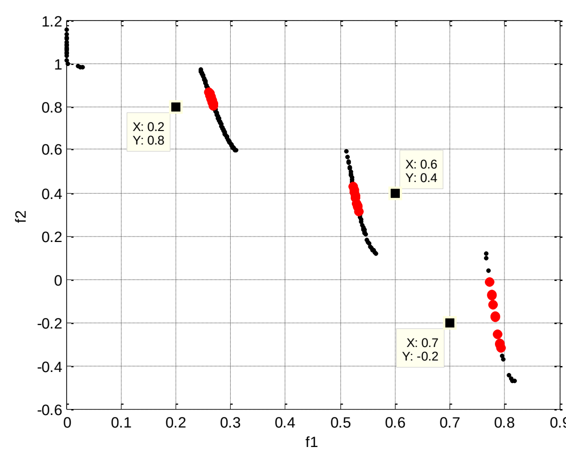

Fig. (4) Pareto Front of M_UC problem using MISA based NN

Test Problem (4). (Two Bar Truss Design)[16]

The industrial problem for optimizing a two bar truss is illustrated in Fig (5). It is comprised of two stationary pinned joints, A and B, where each one is connected to one of the two bars in the truss. The two bars are pinned where the join bars in the truss. The two bars are pinned where the join one another at joint C, and a 100kN force acts directly downward at that point. The cross-sectional areas of the two bars are represented as x_1 and x_2, the cross-sectional areas of trusses AC and BC respectively. Finally, y represents the perpendicular distance from the line AB that contains the two-pinned base joints to the connection of the bars where the force acts (joint C). The stresses in AC and BC should not exceed 100,000kPa and the total volume of material should not exceed $0.1\mathrm{m}^3$. [16]

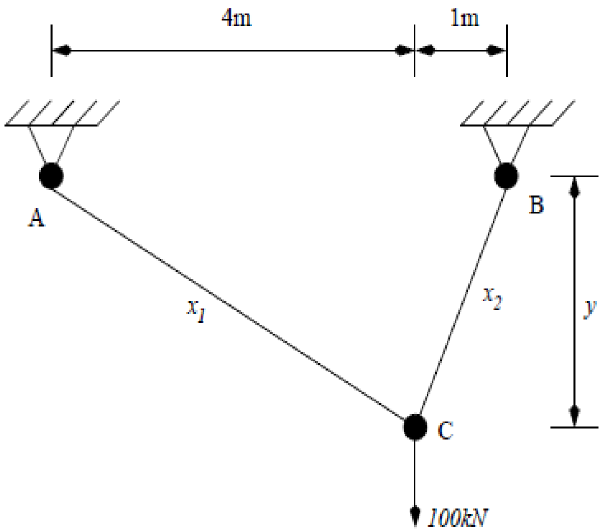

Fig.(5) Two Bar Truss[16]

The problem formulation is:

Minimize $\{f_{volume}, f_{stress\ AC}, f_{stress\ BC}\}$

s.to $1 \le y \le 3$

$x_1, x_2 > 0$

Where $f_{volume} = x_1(16+y^2)^{1/2} + x_2(1+y^2)^{1/2}$

$$f_{stress\ AC} = \frac{20(16+y^2)^{1/2}}{y\,x_1}$$

$$f_{stress\ BC} = \frac{80(1+y^2)^{1/2}}{y\,x_2}$$

The output of NN indicates that the range of variables will $0 \le x_1 \le 1$, $0 \le x_2 \le 1$ and $1 \le y \le 2$. Preferred solutions for three reference points(0,0,0),(0,1e6,0) and (0.005,1e6,4e5), using Reference Point based MMISA based NN shown in fig (6)

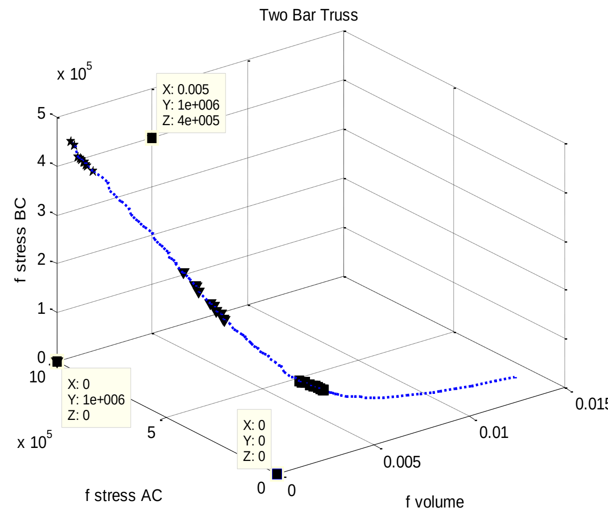

Fig. (6) Preferred solutions for three reference points.

VIII. CONCLUSION

The reference point approach is a common methodology in multi-criterion decision-making, in which one or more reference points are specified by the decision-maker beforehand. The target in such an optimization task is then to identify the Pareto-optimal region closest to the reference points.

We have presented a Modified hybrid Multi-objective optimization algorithm based on the clonal selection principle and Neural Networks. The approach is able to produce results similar or better than those generated by other evolutionary algorithms after determining the max and min values with NN and use it to initialize population with at least feasible antibodies which help MMISA to find the preferred solutions closest to the reference point.

The approach proposed also uses a very simple mechanism to deal with constrained test functions, and our results indicate that such mechanism, despite its simplicity, is effective in practice.

All calculations are carried by **Matlab** 7.2 program, and are run on Laptop 2GHz/ 1Gb RAM/Windows XP, the solution is very fast and take small number of iterations.

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